PRECISE OR IMPRECISE PROBABILITIES?
Evidence from survey response related to late-onset dementia

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Summary

Research measuring expectations has had several complementary objectives:

* To use expectations data to shed light on beliefs and behavior.

* To learn how respondents interpret the questions posed.

* To improve question design, so as to enhance the usefulness of the data.

This paper concerns all three objectives.

Most economic research assumes that agents hold precise subjective probabilities.

Yet economists and others have long been concerned that this may not be the case, especially when information is limited.

This concern has stimulated much research on *imprecise probabilities*, also known as *deep uncertainty* or *ambiguity*.

The research has been largely theoretical or experimental.

Little has been known about the precision of the expectations people hold in real life.

We report findings from an experimental module that we placed in the 2016 HRS.

The standard HRS format asks respondents to report precise probabilities. The data have generally been interpreted at face value.

We start from the precise percent-chance format and then use two probing questions to learn about the nature of reports.

The first probe asks whether the reported probability was intended to be an exact number or was rounded/approximated.

When the response is rounded/approximated, a second probe asks the respondent to give a precise probability or an imprecise probability, stated as a range.

This question format was developed by Manski and Molinari (2010).

We elicited expectations of developing late-onset dementia and related long-term care (LTC) decisions---purchasing LTC insurance or entering a nursing home---among dementia-free respondents in the HRS.

Close to half hold imprecise probabilities, with most interval widths 10 or 20 percent.

Similar fractions express imprecision regarding purchase of LTC insurance and entering a nursing home.

Precision in the LTC question is more common when respondents are given a hypothesized dementia outcome.

Finally, we consider the decision to purchase LTC insurance when persons have imprecise expectations of developing dementia.

Why did we conjecture that expectations may be imprecise?

Available research on the prevalence of dementia does not provide evidence on personalized risk that persons will develop dementia.

Medical researchers have developed online tools that predict the chance that persons with specified attributes will develop cardiovascular disease and breast cancer.

However, there currently exists no similar tool predicting personalized risk of dementia.

The prediction task may be difficult for lay people.

Hence, we conjectured that many persons may hold imprecise expectations of their dementia risk – and also LTC outcomes, which may depend on dementia expectations.

(Im)Precise Probabilities Across Fields

The idea that individuals might not hold precise probability distributions has been investigated from multiple perspectives.

The idea appears early in Keynes (1921) and Knight (1921).

The modern literature on decision under ambiguity was influenced by the thought experiments and argumentation of Ellsberg (1961).

The statisticians Dempster (1968) and Shafer (1976) developed a generalization of the Bayesian theory of subjective probability, featuring upper and lower probabilities. Further work includes Walley (1991)'s *imprecise probabilities* framework.

Philosophers, too, have developed alternatives to strict Bayesianism.

Manski (2000) connects imprecise probabilities with partial identification in econometrics. Empirical research combines assumptions with data to draw conclusions about objective probability distributions. Partial identification occurs when credible assumptions combined with unlimited data drawn by a specified sampling process partially reveals a distribution.

Psychologists have long elicited subjective probabilities. Elicitation of imprecise probabilities was performed by Wallsten, Forsyth, and Budescu (1983).

Manski and Molinari (2010) developed the elicitation format used here, with application by Giustinelli and Pavoni (2017).

Other ongoing work includes Bachman, Carstensen, Lautenbacher, and Schneider (2019), Delavande, Ganguli, and Mendel (2019), and Enke and Graeber (2019).

Our interest derives in part from our previous studies of rounding by HRS respondents. See Manski and Molinari (2010) and Giustinelli, Manski and Molinari (2020).

This research has shed light on the properties of rounding, but the reason why respondents report rounded probabilities has been incompletely understood.

Manski and Molinari (2010) hypothesized that respondents who round their reports might do so to simplify communication and/or to convey partial knowledge.

Our probing questions enable us to distinguish among these possibilities.

Measuring Precise and Imprecise Probabilities

We elicit expectations from 1,293 randomly drawn eligible participants in the 2016 HRS.

Module eligibility required that respondents did not live in a nursing home at the time of the survey and had never been diagnosed with dementia.

Eligible respondents were randomly assigned to one of two question sequences regarding LTC: purchasing LTC insurance or entering a nursing home.

Each was asked their expectations for four outcomes: developing dementia, purchasing insurance (entering nursing home), and purchasing insurance (entering nursing home) if they were to know the true dementia state (not developing dementia, developing dementia).

The analysis of this paper focuses on the question asking respondents to report the chance of developing dementia in the future. The initial question is:

Q0. Dementia is a general term for a decline in mental ability severe enough to interfere with daily life. Memory loss is an example. Alzheimer's is the most common type of dementia. On a scale of 0 to 100, what is the percent chance that you will develop dementia sometime in the future?

When a respondent states that they do not know the chance of the event, the response is recorded as “Don’t know” (DK). If a respondent refuses to answer the question, the response is recorded as “Refuse” (RF).

This expectation question was never asked previously in the HRS nor, as far as we are aware, in any other survey.

We designed three probing sequences, one for each of three types of answer a respondent might give to the initial question:

(i) a numerical point response between 0 and 100 percent

(ii) a numerical interval response

(iii) a DK response.

The standard format in the HRS Core does not allow respondents to answer with an interval. If a respondent spontaneously answers with an interval, the interviewer is instructed to ask them to convert the answer into a single value.

In our module we permitted spontaneous interval responses.

We then asked respondents to convert their initial interval into a point response.

The sequence of follow-up questions after a numerical point response is as follows.

Point FU Q1. When people are asked to give a numerical response, like percent chance, sometimes they give exact answers and sometimes they give rounded or approximate numbers. When you said [X] percent just now, did you mean this as an exact answer or were you rounding or approximating?

Possible answers: Exact answer; Rounding/approximating; Don't know/Refuse.

Respondents indicating that they were rounding were asked a second probing question.

Point FU Q2. Now please try without rounding or approximating your answer. What is the percent chance that [EVENT] sometime in the future? If you are uncertain about the chances, you may give a range. For example, you may say something like "less than 20 percent," "between 30 and 40 percent" or "greater than 80 percent."

Possible answers: percent chance in 0-100; a range; Don't know/Refuse.

Respondents who answered the initial question with a spontaneous interval were asked the following probing question before being routed to the next expectation question.

Interval FU Q1. If you had to answer with a single value to the previous question, what point would you give?

Possible answers: A percent chance in 0-100; Don't know/Refuse.

Respondents who answered the initial question with "Don't Know" were asked the following probing questions.

DK FU Q1. When people are asked to give the percent chance that something will happen in the future, sometimes they give exact answers and sometimes they feel uncertain about the chances. When you said you don't know just now, did you mean you feel uncertain about the chances or something else?

Possible answers: Uncertain about the chances; Something else; Don't know/Refuse.

Respondents indicating that they were uncertain about the chances were asked a second follow-up question; all other respondents were skipped to the next expectation question.

DK FU Q2. If you are uncertain about the chances, you may give a range instead. For example, you may say something like "less than 20 percent," "between 30 and 40 percent" or "greater than 80 percent." If you could give a range, what range would you give to the percent chance that [EVENT] sometime in the future?

Possible answers: A percent chance in 0-100; a range; Don't know/Refuse.

For each initial question, we use the answers to the corresponding probing questions to classify responses into four mutually exclusive and exhaustive categories.

EX: Respondents who gave an exact response are classified as “Exact point probabilities.”

PR: Respondents who gave a rounded/approximated point response followed by an unrounded point response are classified as “Rounded/approximated point probabilities.”

IM: Respondents who gave a rounded/approximated point response followed by an interval are classified as “Interval probabilities.” This category includes respondents who spontaneously answered the initial question with an interval and those who answered DK because they felt unsure about the chances.

Remaining respondents are labelled as “Other.”

Our design elicits precise and imprecise probabilities by means of a probing sequence that follows the standard HRS question, rather than directly giving the respondent the options of reporting the subjective probability as a single value or a range.

We made this choice deliberately. We wanted to use an elicitation procedure that would start from a format with which HRS respondents were familiar, while enabling us to learn about the nature of their reports.

This approach might prompt respondents to think more deeply about the question and, perhaps, to revise their beliefs.

We aimed to choose a neutral wording, to avoid steering respondents toward reporting intervals over unrounded points, or vice versa.

(Im)precision of Subjective Probabilities Related to Dementia

We analyze the responses to the dementia question, focusing first on (im)precision and then on the substantive findings. The sample consists of 1,255 respondents for whom we have complete and logical responses. We drop 38.

We find

Group EX: 35% of respondents.

Group PR: 14% of respondents.

Group IM: 47% of respondents.

Other: 4% of respondents.

Within the two subgroups of Group IM,

28% and 29% have interval width equal to 10 percent.

33% and 21% have interval width equal to 20 percent.

The median interval width is 20 percent in both sub-groups.

The 1st and 9th decile are 10 and 80 percent in both sub-groups.

Some authors have given special attention to the probability responses (0, 50, 10) and have advanced hypotheses about their meaning and interpretation.

Fischhoff and Bruine de Bruin (1999) and Bruine de Bruin et al. (2002) hypothesize that some respondents use 50 percent to signal epistemic uncertainty (extreme imprecision).

Lillard and Willis (2001) and Hudomiet and Willis (2013) conjecture that respondents form precise subjective distributions and then, perhaps to simplify communication, they report whichever of the values (0, 50, 100) is closest to the mode of their distribution.

In our sample, very few respondents initially reported 100%.

Respondents who initially responded 0 percent are

- more likely to report after probing that their response is exact.
- less likely to report that their response is a rounded/approximated point probability.
- less likely to provide a probability interval.

These results are contrary to the modal response hypothesis, which conjectures that responses of 0 percent tend to reflect substantial rounding.

Respondents who initially gave a response of 50 percent are

- less likely than others to report that their response is exact,
- more likely to report that their response is a rounded/approximated point probability.
- as likely to give an interval after probing.

The distribution of interval widths is similar for those who initially answered 50 percent and those who answered with values other than 50.

These results are inconsistent with the epistemic-uncertainty hypothesis, which conjectures that responses of 50 percent tend to reflect substantial imprecision in beliefs.

We investigate how the prevalence of different response types and the amount of imprecision vary with observed respondent characteristics.

Older respondents are more likely than younger ones to report rounded/approximated probabilities and to hold interval probabilities.

More educated respondents are less likely to report a rounded/approximated probability and are more likely to hold imprecise probabilities.

Black respondents are less likely than others to report rounded/approximated probabilities and to hold imprecise probabilities.

We find no association between response type and gender or cognitive ability.

Relationship between Initial and Post-Probe Subjective Probabilities

32% of respondents in Group PR give the same response pre- and post-probe. 36% give a less rounded response and 12% a more rounded one post-probe.

Table 4. Within-Person Comparison of Initial Point Response and Post-Probe Interval Response in Group IM Respondents who first give a point and then an interval

	Any width	Width in: (0, 10]	(10, 20]	(20, 100]
point is midpoint of the interval	11	6	23	3
point is inside interval, not midpoint	58	55	46	74
point is outside interval, distance ≤ 5	4	4	5	3
point is outside interval, distance 6-10	1	19	6	10
point is outside interval, distance > 10	15	16	20	9
N	442	137	157	148

(Im)precision of Long-Term Care Probabilities

Among the 578 respondents who were asked the probability of purchasing LTC insurance, 38% were classified as EX, 15% as PR, 42% as IM, and 5% as Other.

Among the 677 respondents who were asked the probability of entering a nursing home, 34% were classified as EX, 16% as PR, 46% as IM, and 4% as Other.

These distributions are similar to the distribution of response types for dementia.

A similar finding applies to the distributions of interval widths among IM respondents.

When respondents are informed that their risk of dementia is zero or one, the fraction reporting imprecise probabilities decreases substantially.

For those assigned to the LTC insurance question,

- fraction IM decreases from 42% to 25% and 18%.
- fraction PR decreases from 15% to 11% and 10%.
- fraction EX increases from 38% to 61% and 67%.

The results are similar for the nursing home question.

This evidence is consistent with our conceptualization of imprecise probabilities as an expression of limited knowledge or information.

Table 6. Initial and Post-Probe Dementia Probabilities by Probabilistic Response Type

	Initial = Post Probe	*Initial*		*Post* Probe		
Response distribution:	Point-Prob. in EX Group	Point-Prob. in PR Group	Point-Prob. in IM Group	Point-Prob. in PR Group	LB in IM Group	UB in IM Group
1st decile	0	1	5	10	0	20
Median	15	40	30	30	0	40
Mean	25	37	35	34	16	46
9th decile	70	70	70	70	40	100
N	437	180	462	180	444	442

We would like to compare these reported probabilities with the future realized dementia status in our sample of respondents.

Such a comparison will eventually be possible, as the HRS collects data longitudinally.

For now, it is only possible to compare the probabilities with available statistics.

Most available statistics refer to the *prevalence* of dementia at specific points in time.

Prevalence is a different concept from the *lifetime risk* of dementia, conditional on no present dementia.

A small set of estimates of lifetime risk and related statistics in the medical literature analyze data from broadly comparable samples.

Two studies use the Original and/or Offspring cohorts of the Framingham Heart Study (FHS). A third uses the Aging, Demographics and Memory Study (ADAMS).

Using the FHS Original Cohort, Seshadri and Wolf (2007) estimate age- and gender-specific lifetime risk of dementia among FHS participants who were dementia-free at 55. The estimates range between 14% and 24%.

Combining the FHS cohorts, Chene et al. (2015) estimate gender-specific cumulative incidence of dementia for FHS participants who were dementia-free at 65 and/or at 45, adjusted for competing risks of death. The estimates range between 14% and 25%.

Fishman (2017) estimates the probability that a dementia-free person will develop dementia later in life for several starting ages and for two cohorts of ADAMS participants. These estimates range between 24% and 37%.

Implications for Decision Making

Our findings may have implications for prediction and modeling of decisions such as when to retire or whether to purchase LTC insurance.

To illustrate, we give an example concerning purchase of LTC insurance. The example adapts analysis of medical decision making under ambiguity in Manski (2018).

Consider a person deciding whether or not to purchase LTC insurance.

The decision is d, where $d = 1$ if the person buys insurance and $d = 0$ otherwise.

Utility depends on an unknown binary state of nature s. $s = 1$ if the agent develops dementia in the future and $s = 0$ otherwise.

The decision does not affect the probability of developing dementia.

The person prefers being insured in the dementia state and not being insured in the dementia-free state.

Conditional on a choice, utility is higher in the dementia-free state than with dementia.

Thus, state-dependent utility, *U(d, s)*, satisfies the inequalities

$$\begin{cases} U(1,1) > U(0,1) \\ U(1,0) < U(0,0) \\ U(d,0) > U(d,1) \quad \forall d \in \{0,1\}. \end{cases}$$

Let the decision maker have precise probability P_s of developing dementia.

The optimal decision maximizes subjective expected utility.

$$d^* = 1 \quad \text{if} \quad P_s \geq P^* = \frac{U(0,0) - U(1,0)}{[U(1,1) - U(0,1)] + [U(0,0) - U(1,0)]}.$$

Let the decision maker have interval probability, $[P_L, P_H]$. Criteria for decision-making with imprecise probabilities include *maximin* (MM) and *minimax-regret* (MMR).

An MM decision maker evaluates each action by the worst SEU that it might yield and chooses an action with the least-bad worst SEU. The worst feasible SEUs occur when $P_s = P_H$. Hence, the MM choice is $d^* = 1$ if $P_H \geq P^*$.

An MMR decision maker evaluates each action in a given state by the worst reduction in SEU that it might yield relative to the highest SEU achievable. The MMR choice is $d^* = 1$ if $P_M \geq P^*$, where P_M is the midpoint of $[P_L, P_H]$.

Thus, MM and MMR decision makers may make different choices than would one with a precise probability, who maximizes SEU.

Table [8]. Linear Prediction of Probability of Purchasing LTC Insurance by Probability of Developing Dementia

Sample (based on post-probe info):	**ALL Rs (1)**	**EX or PR Dementia Prob (2)**	**IM Dementia Prob (3)**	**EX or PR Dementia Prob (4)**		**IM Dementia Prob (5)**			
IM assumption:						**(5A)** ***maximin***		**(5B)** ***minimax regret***	
Data type:	**Pre-probe**			**Post-probe**					
Outcome	**Probability of LTC Insurance**			**Probability of LTC Insurance**		**Probability of LTC Insurance**		**Probability of LTC Insurance**	
Predictors				**LB**	**UB**	**LB**	**UB**	**LB**	**UB**
Dementia Prob	0.08 (0.05)	0.15 (0.06)	-0.02 (0.08)	0.09 (0.06)	0.11 (0.07)	0.12 (0.07)	0.19 (0.08)	0.08 (0.08)	0.12 (0.10)
Intercept	20.46 (2.00)	18.94 (2.47)	23.56 (3.42)	16.84 (2.29)	26.24 (2.81)	9.95 (3.13)	21.33 (3.76)	12.92 (2.83)	26.21 (3.46)

Conclusion

Looking forward, we view the work in this paper as a natural next step advancing the research program on measuring expectations.

Until the early 1990s, there were no large scale household surveys querying respondents about their subjective expectations in a probabilistic manner.

In the 1990s, economists began collecting numerical probabilistic expectations data.

The common practice has been to treat responses as precise probabilities.

We think it natural to explore imprecision in beliefs. The elicitation mechanism utilized in this paper is a concrete proposal for doing so.

We view our work as contributing to a new phase of measuring expectations, with important questions still to be addressed.

Considerations of survey space emerge immediately: the probing questions in our elicitation module add length to the HRS instrument.

While the empirical evidence suggests that our sequence of probes successfully elicits imprecise probabilities, more effective question formats might be possible.

For example, a survey could directly offer to respondents the possibility to report their beliefs as intervals.

It is also an open question whether imprecise probabilities should be elicited in all contexts, or only in those where imprecision seems a prominent concern.